- Quiz 1 is open-notes and consists of three questions. There are 15 points available. Take a quick scan through the questions first and prioritize your time accordingly.

- Show all of your work for full marks, and ensure your notation is legible, correct, and consistent with that used in the course.

- Upload clear, legible photos/screenshots of your handwritten answers to the questions within the time window, one question at a time.

- You may refer to the lecture slides. If you need to use a result from lecture, either refer to it by its number or its name (if it is numbered/named), or describe the result.

Good luck!

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1. (5 points) Let $X_1, X_2, \ldots, X_n$ be a random sample from a continuous distribution with pdf

$$f_\alpha(x) = \frac{\Gamma(\alpha + \frac{1}{2})}{\pi^{3/2}\Gamma(\alpha)} 2x \sin(x)^{2\alpha-1}, \qquad x \in (0, \pi), \quad \alpha > 1.$$

Let $T(\mathbf{X}) = \prod_{i=1}^{n} \sin(X_i)$.

(a) (2 points) Show that $T(\mathbf{X})$ is a sufficient statistic for α.

(b) (3 points) Show that $T(\mathbf{X})$ is a complete statistic.

2. (5 points) Answer each of the following questions by writing YES or NO (1 point), and justify your answer in *at most* three sentences (1.5 points).

   (a) Let $T(\mathbf{X})$ be a sufficient statistic, and suppose that $q(\cdot)$ is any function of T. Is it true that $q(T(\mathbf{X}))$ is also a sufficient statistic?

   (b) Suppose that $S = S(\mathbf{X})$ is continuous and ancillary for θ. Does this imply that all of the moments of S (i.e., $\mathbb{E}[S], \mathbb{E}[S^2], \mathbb{E}[S^3] \ldots$) are free of θ?

3. (5 points) Let X have a continuous distribution in an exponential family, and suppose that $r(\cdot)$ is one-to-one and continuously differentiable on the support of X. Prove that the distribution of $r(X)$ is also in an exponential family.